# Principal component analysis of skin color and its application to colorimetric color reproduction on CRT display and hardcopy

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## Abstract

In this paper, we propose a new method to predict the images on a CRT and a hardcopy of skin color under various illuminants. Spectral reflectance of human skin is analyzed by principal component analysis and it is shown that the spectral reflectance can be estimated by three basis functions. The estimation allows colorimetric color reproduction without colorimetric measurements for each illuminant. The proposed method is verified by several skin color patches taken by a calibrated HDTV camera. The method is also applied to practical facial pattern images.

### Introduction

The appearance of skin color is depending on the illuminants in the environment. In the development of cosmetics and their sales promotion, it is required to predict the skin color images under various illuminants. Practically, the skin color images taken under only one illuminant should be transformed to be skin color images taken under various illuminants. In conventional color reproduction techniques<sup>1</sup>, however, it is difficult to reproduce a skin color under various illuminants.

In the conventional color reproduction, color patches under the illuminants are measured colorimetrically to match the color between an illuminated object and an image reproduced by an output device. The result of colorimetric measurements should be analyzed to make a LUT (look up table) or transform operation for each illuminant. The repetition of such process for each illuminant would be a restraint for the development of cosmetics and their sales promotion.

A previous work shows that the spectral reflectance of human skin can be represented by three basis functions based on the principal component analysis.<sup>2</sup> Then, the two dimensional distribution of spectral reflectance

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can be estimated from the values of three color channels and the spectral radiance of the illuminant, as is performed in electronic endoscope images.<sup>3</sup>

In this paper, we propose a new skin color reproduction method to predict the skin color images under various illuminants. The proposed method uses estimated spectral reflectance. The estimation will allow colorimetric reproduction without colorimetric measurements for each illuminant. Those processed skin colors are reproduced both on a CRT display and a hardcopy.

The principal component analysis of the spectral reflectance and a method to estimate the spectral reflectance are described. The proposed method is verified by several skin color patches taken by a calibrated HDTV camera and the method is applied to facial pattern image.

### **Principal Component Analysis of Spectral Reflectance of Human Skin**

Ojima et al.<sup>2</sup> measured one hundred eight spectral reflectance of skin in human face for 54 Mongolians (Japanese women) who are between 20 and 50 years old. The Munsell values of the sample have a range as follows; H=2YR-8YR, V=5-7, C=2-5, and the distribution of these skin colors in CIE 1976 L\*a\*b\* color space is shown in Fig. 1. The spectral reflectance was measured at intervals of 5 nm between 400 nm and 700 nm. Therefore, the spectral reflectance is described as vectors o in 61dimensional vector space. Figure 2 shows the averaged spectral reflectance of human skin. The covariance matrix of the spectral reflectance was calculated for the principal component analysis. The eigenvectors of the covariance matrix are named as principle component vectors. Then, the spectral reflectance of human skin can be expressed as a linear combination of the principle component vectors as follows:

$$\boldsymbol{o} = \bar{\boldsymbol{o}} + \sum_{i=1}^{n} \boldsymbol{\alpha}_{i} \boldsymbol{u}_{i} , \qquad (1)$$

where  $\overline{o}$  is the averaged spectral reflectance,  $u_i$  (i=1...n) are the eigenvectors and  $\alpha_i$  (i=1...n) are the eigenvalues. The eigenvectors are combined in order of the eigenvalues.

The cumulative proportion rates of the principle component vectors are shown in Fig. 3. From this figure, we can see that the cumulative proportion rate from first to third components is about 99.5%. Then, the spectral reflectance of human skin can be represented approximately 99.5% by using linear combination of three principal components  $u_1, u_2, u_3$ . The three principal components are shown in Fig. 4. Therefore the Equation (1) can be represented approximately as follows:

$$\boldsymbol{o} \cong \overline{\boldsymbol{o}} + \sum_{i=1}^{3} \boldsymbol{\alpha}_{i} \boldsymbol{u}_{i} = \overline{\boldsymbol{o}} + \begin{pmatrix} \boldsymbol{u}_{1} & \boldsymbol{u}_{2} & \boldsymbol{u}_{3} \end{pmatrix} \begin{pmatrix} \boldsymbol{\alpha}_{1} \\ \boldsymbol{\alpha}_{2} \\ \boldsymbol{\alpha}_{3} \end{pmatrix}.$$
(2)

### **Estimation of the Spectral Reflectance of Human Skin**

Considering the result of above principal component analysis, we can estimate the spectral reflectance of skin using the tristimulus values of the human skin. The tristimulus values can be easily measured by a colorimeter. The spectral reflectance of skin is estimated as follows:

As well known, the tristimulus values *X*, *Y*, *Z* can be calculated by Eq. (3).

$$X = K \sum_{\lambda = 400}^{700} E(\lambda) \overline{x(\lambda)} O(\lambda)$$
  

$$Y = K \sum_{\lambda = 400}^{700} E(\lambda) \overline{y(\lambda)} O(\lambda),$$
  

$$Z = K \sum_{\lambda = 400}^{700} E(\lambda) \overline{z(\lambda)} O(\lambda)$$
(3)

where  $O(\lambda)$  is the spectral reflectance,  $E(\lambda)$  is the spectral radiance of the illuminant,  $\overline{x(\lambda)}$ ,  $\overline{y(\lambda)}$ ,  $\overline{z(\lambda)}$  are color matching functions and K is a constant. By vector notations, Eq. (3) can be expressed as follows:

$$X = Ke' \overline{X} o$$

$$Y = Ke' \overline{Y} o ,$$

$$Z = Ke' \overline{Z} o$$
(4)

where  $[.]^{t}$  represents the transpose of [.], the vectors e, o are vector notations of  $E(\lambda)$  and  $O(\lambda)$  respectively, and the matrixes  $\overline{X}$ ,  $\overline{Y}$ ,  $\overline{Z}$  are represented as follows:

$$\bar{x}(\lambda) \rightarrow \bar{X} = \begin{pmatrix} \bar{x}_{1} & & O \\ & \bar{x}_{2} & & \\ O & & \bar{x}_{n} \end{pmatrix}, \qquad (5)$$
$$\bar{y}(\lambda) \rightarrow \bar{Y} = \begin{pmatrix} \bar{y}_{1} & & O \\ & \bar{y}_{2} & & \\ O & & \bar{y}_{n} \end{pmatrix}, \qquad (6)$$
$$\bar{z}(\lambda) \rightarrow \bar{Z} = \begin{pmatrix} \bar{z}_{1} & & O \\ & \bar{z}_{2} & & \\ O & & \bar{z}_{n} \end{pmatrix}. \qquad (7)$$

From Eq. (2), the Eq. (4) can be written as,

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} \cong Ke^{t} \begin{pmatrix} \overline{X} \\ \overline{Y} \\ \overline{Z} \end{pmatrix} \begin{bmatrix} \overline{o} + (u_{1} \quad u_{2} \quad u_{3}) \begin{pmatrix} \alpha_{1} \\ \alpha_{2} \\ \alpha_{3} \end{pmatrix} \end{bmatrix}.$$
(8)

The Equation (8) can be rewritten as follows:

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} \cong K \boldsymbol{e}^{t} \begin{pmatrix} \overline{\boldsymbol{X}} \\ \overline{\boldsymbol{Y}} \\ \overline{\boldsymbol{Z}} \end{pmatrix} \boldsymbol{\bar{o}} + K \boldsymbol{e}^{t} \begin{pmatrix} \overline{\boldsymbol{X}} \boldsymbol{u}_{1} & \overline{\boldsymbol{X}} \boldsymbol{u}_{2} & \overline{\boldsymbol{X}} \boldsymbol{u}_{3} \\ \overline{\boldsymbol{Y}} \boldsymbol{u}_{1} & \overline{\boldsymbol{Y}} \boldsymbol{u}_{2} & \overline{\boldsymbol{Y}} \boldsymbol{u}_{3} \\ \overline{\boldsymbol{Z}} \boldsymbol{u}_{1} & \overline{\boldsymbol{Z}} \boldsymbol{u}_{2} & \overline{\boldsymbol{Z}} \boldsymbol{u}_{3} \end{pmatrix} \begin{pmatrix} \boldsymbol{\alpha}_{1} \\ \boldsymbol{\alpha}_{2} \\ \boldsymbol{\alpha}_{3} \end{pmatrix}.$$
(9)

We can consider the first term of Eq. (9) as a contribution of the averaged spectral reflectance to the tristimulus values, and the second term as a contribution of three eigenvectors. Then, we can rewrite the Eq. (9) as follows:

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} \cong \begin{pmatrix} \overline{X} \\ \overline{Y} \\ \overline{Z} \end{pmatrix} + \begin{pmatrix} X_1 & X_2 & X_3 \\ Y_1 & Y_2 & Y_3 \\ Z_1 & Z_2 & Z_3 \end{pmatrix} \begin{pmatrix} \boldsymbol{\alpha}_1 \\ \boldsymbol{\alpha}_2 \\ \boldsymbol{\alpha}_3 \end{pmatrix},$$
(10)

where  $\overline{X}$ ,  $\overline{Y}$ ,  $\overline{Z}$  are the averaged tristimulus values and  $X_i$ ,  $Y_i$ ,  $Z_i$ (*i*=1,2,3) are the tristimulus values corresponding to the three eigenvectors of spectral reflectance of skin.

Then, the eigenvalues  $\alpha_1, \alpha_2$ , and  $\alpha_3$  are given by

$$\begin{pmatrix} \boldsymbol{\alpha}_{1} \\ \boldsymbol{\alpha}_{2} \\ \boldsymbol{\alpha}_{3} \end{pmatrix} \cong \begin{pmatrix} X_{1} & X_{2} & X_{3} \\ Y_{1} & Y_{2} & Y_{3} \\ Z_{1} & Z_{2} & Z_{3} \end{pmatrix}^{-1} \begin{bmatrix} X \\ Y \\ Z \end{bmatrix} - \begin{pmatrix} \overline{X} \\ \overline{Y} \\ \overline{Z} \end{bmatrix} \end{bmatrix}.$$
(11)

The spectral reflectance of human skin can be estimated by above eigenvalues and three principal components by Eq. (2).

# Two Dimensional Measurement of Spectral Reflectance by HDTV Camera

We measured the tristimulus values X, Y, Z of a human skin in each pixel by a HDTV camera. A two-dimensional distribution of spectral reflectances of human face can be estimated from the measured tristimulus values as described above. Output values R,G,B by HDTV camera was transformed to X, Y, Z as follows:<sup>2</sup>

$$R = K_r f_r(R_o)$$
  

$$G = K_g f_g(G_o),$$
  

$$B = K_b f_b(B_o)$$
  
(12)

$$R_{o} = \sum_{\lambda=400}^{700} E(\lambda) \overline{r(\lambda)} O(\lambda)$$

$$G_{o} = \sum_{\lambda=400}^{700} E(\lambda) \overline{g(\lambda)} O(\lambda),$$

$$B_{o} = \sum_{\lambda=400}^{700} E(\lambda) \overline{b(\lambda)} O(\lambda)$$
(13)

where  $\overline{r(\lambda)}$ ,  $\overline{g(\lambda)}$ ,  $\overline{b(\lambda)}$  are the spectral sensitivity functions of the camera,  $f_r$ ,  $f_g$ ,  $f_b$  are non-linear functions, and  $K_r$ ,  $K_g$   $K_b$  are white balance constants. The non-linearity between RGB level and luminance was compensated to be linearity by using the following quadratic equations:

$$R' = -5.50 + 4.26 \times 10^{-1} R + 2.04 \times 10^{-3} R^{2}$$

$$G' = -6.06 \times 10^{-1} + 2.90 \times 10^{-1} G + 2.66 \times 10^{-3} G^{2},$$

$$B' = -7.37 \times 10^{-1} + 2.31 \times 10^{-1} B + 3.11 \times 10^{-3} B^{2}$$
(14)

where R', G', B' are compensated values. The compensated values R', G', B' were transformed to the tristimulus values X, Y, Z by Eq. (15).

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = M_1 \begin{pmatrix} R \\ G' \\ B' \\ 1 \end{pmatrix}, \qquad (15)$$

where  $M_1$  is a 3x4 matrix. This matrix  $M_1$  can be determined by multiple regression analysis. Thirty-nine patches of Japanese skin color were used at the multiple regression analysis. The Munsell values of the patches have a range as follows; H=0YR~10YR, V=5~8, C=2~5. The pictures were taken under standard illuminant C at 2° field of view by the HDTV camera. The tristimulus values of the patches were measured under standard illuminant C by a spectrophotometer (Minolta CM1000). In CIE 1976 L\*a\*b\* color space, the averaged color difference and the maximum color difference were calculated between the measured and calculated tristimulus values. The averaged color difference  $\overline{\Delta E}_{L^*a^*b^*}$  was 0.99 and the maximum color difference  $\Delta E_{L^*a^*b^*}^{Max}$  was 2.30. This result shows that the transformation has sufficient accuracy to estimate the spectral reflectance by Eq. (11).

# **Skin Color Reproduction Based on Spectral Reflectance**

Figure 5 shows schematic diagram of color reproduction from an image taken by HDTV camera to CRT or hardcopy of skin color under various illuminants. The tristimulus values X', Y', Z' of skin color under a selected illuminant are easily calculated from the estimated spectral reflectances  $O(\lambda)$  and the spectral radiance  $E_2(\lambda)$  of the illuminant.

Color reproduction on CRT display. The tristimulus values X', Y', Z' are transformed to input levels  $R_c$ ,  $G_c$ ,  $B_c$  of CRT display using the transform operation  $M_2$ . The transform operation  $M_2$  was measured as described below<sup>4</sup>.

At first, the relationship between input level  $R_c$ ,  $G_c$ ,  $B_c$  and luminance of display phosphor was analyzed. A CRT display (Nanao Flex Scan56T Monitor) with fixed luminance, contrast, white point, and gamma were used in the experiment. Twenty-six color patches were displayed on the monitor in a dark environment to avoid interference by external light sources. The luminance *L* and the chromaticitiy *x*, *y* of the displayed color patches in each channel were measured by a luminance colorimeter (TOPCOM BM-7). The relationship between the input level and luminance is plotted in Fig. 6. The relationship between the luminance and input levels is

$$L_{R} = a_{0}R_{c}^{2} + a_{1}R_{c} + a_{2}$$

$$L_{G} = b_{0}G_{c}^{2} + b_{1}G_{c} + b_{2},$$

$$L_{B} = c_{0}B_{c}^{2} + c_{1}B_{c} + c_{2}$$
(16)

where  $L_{R}$ ,  $L_{G}$   $L_{B}$  are the luminance of red, green, and blue phosphors respectively, and  $a_{i}$ ,  $b_{i}$ ,  $c_{i}$  (i=0 to 2) are coefficients.

The tristimulus values X, Y, Z on the display can be decomposed to R, G, B contribution terms as shown in the following equation:

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \begin{pmatrix} X_{R} + X_{G} + X_{B} \\ X_{R} + Y_{G} + Y_{B} \\ Z_{R} + Z_{G} + Z_{B} \end{pmatrix},$$
(17)

where  $X_i$ ,  $Y_i$ ,  $Z_i$  (*i=R*, *G*, *B*) are the tristimulus values corresponding to the emission of red, green, blue phosphor, respectively. The tristimulus values corresponding to each phosphor are calculated from the measured L, x and y. A relationship between X-Y, Z-Y for each phosphor is shown in Fig. 7. These relations can be represented by a linear equation as follows:

$$X_{R} = a_{R}Y_{R} + b_{R}$$

$$X_{G} = a_{G}Y_{G} + b_{G}$$

$$X_{B} = a_{B}Y_{B} + b_{B}$$

$$Z_{R} = c_{R}Y_{R} + d_{R}$$

$$Z_{G} = c_{G}Y_{G} + d_{G}$$

$$Z_{B} = c_{B}Y_{B} + d_{B}$$
(18)

where  $a_i, b_i, c_i, d_i$  (*i=R, G, B*) are coefficients.

From Eqs. (17) and (18), the following equation is obtained.

$$\begin{pmatrix} X \\ Y \\ Z \end{pmatrix} = \begin{pmatrix} a_{R}Y_{R} + a_{G}Y_{G} + a_{B}Y_{B} + b_{R} + b_{G} + b_{B} \\ Y_{R} + Y_{G} + Y_{B} \\ c_{R}Y_{R} + c_{G}Y_{G} + c_{B}Y_{B} + d_{R} + d_{G} + d_{B} \end{pmatrix} = A \begin{pmatrix} Y_{R} \\ Y_{G} \\ Y_{B} \end{pmatrix} + \begin{pmatrix} b_{R} + b_{G} + b_{B} \\ 0.0 \\ d_{R} + d_{G} + d_{B} \end{pmatrix},$$
$$A = \begin{pmatrix} a_{R} & a_{G} & a_{B} \\ 1.0 & 1.0 & 1.0 \\ c_{R} & c_{G} & c_{B} \end{pmatrix}.$$
(19)

The luminance can be calculated from the Eq. (19) as follows,

$$\begin{pmatrix} L_{R} \\ L_{G} \\ L_{B} \end{pmatrix} = \begin{pmatrix} Y_{R} \\ Y_{G} \\ Y_{B} \end{pmatrix} = A^{-1} \begin{pmatrix} X - b_{R} - b_{G} - b_{B} \\ Y \\ Z - d_{R} - d_{G} - d_{B} \end{pmatrix}.$$
 (20)

Then, using the Eqs. (16) and (20), the transformation from the tristimulus values X, Y, Z to the input levels  $R_c$ ,  $G_c$ ,  $B_c$  can be achieved.

**Color Reproduction in Hardcopy.** As shown in Fig. 5, tristimulus values X'', Y'', Z'' of a printed skin image under a certain illuminant were matched to the tristimulus values X', Y', Z' of the same skin image on a CRT display. To achieve the above matching, the matrix  $M_3$  was used to transform the CRT input levels  $R_c$ ,  $G_c$ ,  $B_c$  to printer input levels  $R_p$ ,  $G_p$ ,  $B_p$ . The matrix  $M_3$  depends on the spectral radiance  $E_3(\lambda)$  of the illuminant on the hardcopy. As shown in Fig. 8 the matrix  $M_3$  was calculated by multiple regression analysis using the measured spectral radiance  $E_3(\lambda)$  and spectral reflectance of color patches. It is noted that we need not to measure the tristimulus values of the color patches under each illuminant.

One hundred eight skin color patches with printer input level  $R_p^n$ ,  $G_p^n$ ,  $B_p^n$  were printed using Fujix Pictrography 3000. The spectral reflectance  $O^n(\lambda)$  of each patch was measured by a spectrophotometer (Datacolor Spectraflash 500).

In printing, the tristimulus values  $X^{n}$ ,  $Y^{n}$ ,  $Z^{n}$  are calculated under a selected illuminant  $E_{3}(\lambda)$  using the available spectral reflectances  $O^{n}(\lambda)$  by the transformation matrix  $M_{2}$ . The tristimulus values  $X^{n}$ ,  $Y^{n}$ ,  $Z^{n}$ .

for each patch were transformed to  $R_c^n$ ,  $G_c^n$ ,  $B_c^n$  which are input levels of CRT. In the Eq. (21), the coefficients ( $a_{i,j}$ ) (*i*=1...3, *j*=1...11) of transformation matrix  $M_3$  were determined by multiple regression analysis.

$$\begin{pmatrix} R_{p} \\ G_{p} \\ B_{p} \end{pmatrix} = \begin{pmatrix} a_{1,1} & a_{1,2} & \dots & a_{1,11} \\ a_{2,1} & a_{2,2} & \dots & a_{2,11} \\ a_{3,1} & a_{3,2} & \dots & a_{3,11} \end{pmatrix} \begin{pmatrix} R_{c} \\ G_{c} \\ B_{c} \\ B_{c}^{2} \\ B_{c}^{2} \\ R_{c} G_{c} \\ G_{c} B_{c} \\ R_{c} B_{c} \\ R_{c} G_{c} B_{c} \\ I \end{pmatrix}$$
(21)

The accuracy of this colorimetric color reproduction was evaluated by average color differences of fifty five skin color patches used in the multiple regression analysis. Namely color difference was calculated between CRT and hardcopy, with and without color transformation by  $M_3$ . As shown in Fig. 9, the averaged color difference  $\overline{\Delta E}_{L^*a^*b^*}$  was 19.85 without the color transformation, and 4.45 with the color transformation. Thereafter, fifty five skin color patches not used in the multiple regression analysis were printed with color transformation by  $M_3$ . The averaged color difference  $\overline{\Delta E}_{L^*a^*b^*}$  was 4.89. We can conclude that the proposed color transformation is effective to match the skin color between displayed image and hardcopy. The average calculation time of multiple regression analysis for matrix  $M_3$  was 25 seconds in a workstation (sparc station II; Sun micro system Inc.).

### **Experiments and Discussion**

Five portrait images with 1920 by 1035 pixels were taken by a HDTV camera under illuminant C. The model is a Japanese young woman. Four kinds of illuminants; "Day light" (CIE D65), "A" (2,856 K), "Cool white" (4,150 K) and "Horizon" (2,300 K) in a standard illumination booth (Macbeth Spectralight II) were used in the experiment. The spectral radiance from 400 nm to 700 nm of each illuminant was measured by a spectroradiometer as shown in Fig. 10. The predicted images displayed on CRT and hardcopys were observed.

We printed directly the portrait images with the input levels  $R_c$ ,  $G_c$ ,  $B_c$  of the CRT display. The printed images are shown in Fig. 11. We can see that the portrait images under "A" and "Horizon" illuminants seem reddish, because longer wavelength components predominate in these illuminants as shown in Fig. 10. These show that our color reproduction system works well.

In case of hardcopy, the portrait images printed using transformation matrix  $M_3$  are shown in Fig. 12. The printed images under "A" and

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"Horizon" illuminants are not reddish as the corresponding portrait images in Fig. 11, because we are not observing the images under each illuminant. Here, it is noted that colors of printed portrait images in Fig. 12 are slightly different. It is explained by the fact that the spectral reflectance of human skin and the spectral reflectance of the printed skin color are different. Averaged spectral reflectance of printed skin color is shown in Fig. 13. From Fig. 2 and Fig. 13, we can see the differences of the spectral reflectance between human skin color and printed skin color.

We can conclude that our color reproduction system can make a colorimetric color reproduction of portrait images on CRT monitor and hardcopy. However, the perceived colors were different in practice. For example, the perceived colors on CRT display were more reddish than perceived skin color, because we are not considering chromatic adaptation in the reproduction of portrait images on CRT monitor.

### **Conclusion and Future Work**

In this paper, we colorimetrically predicted skin color images under various illuminants on both the CRT display and hardcopy. The color reproduction is based on the estimation of spectral reflectance of skin. The two dimensional estimation of spectral reflectance allowed colorimetric reproduction without colorimetric measurements for each illuminant.

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The color reproduction described in this paper is not applicable to the lips, hair, eyes and so on, because only the spectral reflectance of human skin was considered here. The proposed reproduction method will be improved by using their spectral reflectances. Furthermore research considering color appearance models such as von Kries model<sup>5</sup> should be also performed.

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