### Point Spread Function of Specular Reflection and Gonio-Reflectance Distribution<sup>1</sup>

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Abstract. In this article, the specular reflection phenomenon is analyzed by the concept of the point spread function (PSF). The conventional PSF, which is a sharpness criterion, is used for the transfer function in image science. In general, print (an ink image on paper) is observed as diffuse reflection light, and the light source image, i.e., gloss, is observed as specular reflection. We first introduce the point spread function of specular reflection (SR-PSF). A measurement principle for the SR-PSF is proposed, and a measurement apparatus for the SR-PSF is developed by using a collimator optical system. The experimental results show that the measured SR-PSF works as a transfer function for a specular reflection image. Each detected position is in agreement with the angle of reflected light. The apparatus works as a goniophotometer within a narrow solid angle. The relation between the SR-PSF and the specular reflection angle distribution is discussed in terms of surface geometry. It is concluded that the measured data are the SR-PSF and they have a gonio-reflectance distribution. The reflection angle is caused by the surface normal, and the surface normal distribution can be calculated from the gonio-reflectance distribution by using the mathematical model. We plan to apply our proposed principle to the development of an evaluation technique for the visual gloss. In addition, the high-resolution goniophotometer can be further developed by this measuring technique. © 2015 Society for Imaging Science and Technology.

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### INTRODUCTION

Gloss is one of the important qualities for printing paper. Many industrial printing papers are classified into glossy paper or matte paper. When a printed image is observed under a light source, gloss on the surface is observed in addition to the printed image. In a dichromatic reflection model, the intensity of the reflected light is the sum of the diffuse reflection and the specular reflection. As shown in Figure 1, part of the incident light is absorbed, scattered, and widely reflected in all directions. This is called diffuse reflection. The printed image is observed as a diffuse reflection phenomenon. Specular reflection is the mirror-like reflection of light from a surface. Specular reflection is a much more directional reflection. The physical nature of gloss is a specular reflection phenomenon.

The surface normal, shown in Figure 2, indicates the direction of the surface. It can be calculated from the incident light angle and the specular reflection angle. It is assumed that a surface consists of many small facets. Each facet has a surface normal and each reflects light in one direction. The variation in surface normals is the cause of the variation of specular reflection around the center of a mirror reflection angle. The reflectance at the deviation angle from the specular reflection angle is named gonio-reflectance, which can be measured by a goniophotometer. As shown in Figure 3,<sup>1</sup> the goniophotometer has a movable detector to measure the reflectance at different angles.

The standard generally used to evaluate the gloss of paper is set by the international organization for standardization (ISO). However, in some cases, the visual gloss is not in agreement with the ISO evaluation of specular gloss. As shown in Figure 4, a visual test of the sharpness of a specular reflection image on the surface of paper is often performed to estimate the gloss of paper. If the specular reflection image is sharp, we estimate that it has high gloss. This means that gloss can be estimated by measuring the sharpness of the specular reflection image. We considered that the point spread function (PSF) theory, which estimates the sharpness of an image, could be applied to analyze gloss.

The conventional PSF, which is a sharpness criterion, is used for the transfer function of images in image science.<sup>2</sup> The conventional PSF has been studied for transparent objects such as films or lenses. The modulation transfer function (MTF) and PSF are a set of Fourier transforms and can be converted from one to the other. In respect of the diffuse reflection phenomenon, studies of the MTF of paper have previously reported.<sup>3,4</sup>

In the specular reflection phenomenon, there is a technology similar to the MTF and it is the distinctness-ofimage (DOI, this quality is sometimes called Image Clarity, ASTM D5767,<sup>5</sup> JIS K7374<sup>6</sup>). The DOI gloss is widely used to estimate the perceived distinctness and sharpness of the observed mirror image after reflection from the surface.<sup>7</sup> The DOI is defined by JIS K7374 in Japan. The study reported it

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Figure 1. Schematic diagram of a dichromatic reflection model.



Figure 2. Schematic diagram of facets and the surface normal.

could estimate the visual gloss of high gloss ink jet papers.<sup>6</sup> However, low sharpness material like printing paper can hardly be evaluated by the DOI. One of the authors reported that the DOI (JIS K7374) cannot be used to estimate printing paper properties.<sup>8,9</sup> The universal method to describe the physical reflectance properties is through the bidirectional reflectance distribution function (BRDF).<sup>7,10,11</sup> It is thought the BRDF is one of the gonio-reflectance distributions. However, the measurement of the BRDF in and around the specular direction is not easy to perform. The direct comparison of BRDF measurements results in different measuring methods and becomes complicated in the absence of standardization.<sup>7</sup>

In this article, we propose the point spread function of specular reflection (SR-PSF) and develop an SR-PSF measurement apparatus based on the PSF theory. The SR-PSFs of sample papers are measured in experiments, and it is confirmed that the proposed SR-PSF works as a transfer function of a specular reflection image. Furthermore, we show that this apparatus works as a goniophotometer within a narrow solid angle. In experiments, it is also confirmed that the reflection angle of light can be measured by this apparatus. The specular reflection angle is mainly determined by inclination of the surface. Therefore, the surface normal can be determined by the incident angle and the reflection angle. The surface normal distribution expresses the surface geometry of the object. These pieces of information are needed by many reflection models in computer graphics. The relation between the SR-PSF and the specular reflection angle is discussed on the basis of the surface geometry. Additionally, the scientific and technical merits of the SR-PSF are discussed.



Figure 3. Schematic diagram of goniophotometer and gonio-reflectance.



Figure 4. Photographs showing the visual inspection of paper gloss. The upper right is high gloss paper, the middle one is glossy, and the bottom one is matte paper.

### THEORY

### Point Spread Function of Specular Reflection (SR-PSF)

The PSF describes the response of an imaging system to a point source or point object. A more general term for the PSF is the impulse response of the imaging system. Figure 5(a) shows one of the measurement principles of the conventional PSF for an optical system. The point light source is made into parallel light with a collimator. The parallel light passes through the object for the measurement. The parallel light becomes a focused image with another collimator. The spread of the measured point image is defined as the PSF.



Figure 5. Schematic diagram of different measurement principles. (a) Measurement principle for conventional PSF. (b) Proposed measurement principle for SR-PSF.





**Figure 6.** Schematic diagram of collimator optical system. Focal point distance, d, can be calculated from focal length, f, and incident light angle,  $\Delta \theta$ .

We applied this conventional PSF measurement principle to the specular reflection phenomenon. The conventional method was improved to measure the specular reflection, as shown in Fig. 5(b). The point light source is made into parallel light with a collimator. The parallel light is reflected on the measurement object. The reflected light becomes a focused image with another collimator. The spread of the measured point image is defined as the SR-PSF. The SR-PSF is defined as an impulse response of specular reflection.

#### Characteristics of the Collimator Optical System

In this study, we use a collimator optical system for the measurement of the SR-PSF. In the collimator optical system, the focal position is determined by the angle of the input light. The collimator optical system has a focus on one side of the lens, and on the other side is parallel light. The focal position is determined by the angle of the parallel light, which is calculated by the position of the focal plane.

The schematic diagram of a collimator optical system is shown in Figure 6. The distance from the center, d, is calculated with the incident light angle,  $\Delta \theta$ , and the focal



Figure 7. Diagram of apparatus for measuring SR-PSF.



Figure 8. Photograph of the measurement apparatus for SR-PSF.

length, f, in Eq. (1).

$$d = f \cdot \sin(\Delta \theta). \tag{1}$$

It is thought that this feature of the collimator optical system can be used in the analysis of the reflection angle distribution.

### EXPERIMENTS AND RESULTS Developed Apparatus for SR-PSF

We developed an SR-PSF measurement apparatus based on the PSF theory. Figures 7 and 8 show the apparatus used to measure the SR-PSF in this article. The light source and the observation angles are set to 75°. The pinhole is made of a metal plate, and its diameter is 0.1 mm. The focal length, f, (shown in Fig. 6) is 50.1 mm in the collimator optical system. The image resolution of the CCD camera is  $512 \times 512$  pixels, and it has a 16-bit output level per pixel. The pitch of one pixel corresponds to 0.029 mm on the object plane. The output values can be used as the light intensity because the linearity between the output values and the light intensity was confirmed in advance. The sample paper is set on the sample bed, and the SR-PSF is measured in a darkroom. We prepared and measured a black glass whose refractive index is 1.567 to perform the calibration process for the measured values.

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### Direction with respect to the incidence plane

Figure 9. Typical measurement result of SR-PSF. The left figure shows images, the center figure shows the cross-sections, and the right figure shows 3D meshes.



Figure 10. Measurement results of SR-PSFs. Note: the maximum value of the intensity (z-axis) is 60,000 in (a), 6000 in (b), and 500 in (c).

Sample	ISO 75° Specular gloss
Black glass	94.6
Ink jet paper	79.4
Coated paper No. 1	80.1
Coated paper No. 2	64.0
Coated paper No. 3	50.5
Coated paper No. 4	21.7

Table I. Samples and ISO 75° specular gloss.

The developed apparatus is similar to the measurement apparatus of the DOI ("Test method B" in ASTM D5767, or JIS K7374). We think the purpose of the DOI is to measure the MTF of specular reflection. The purpose of the developed apparatus is to measure the PSF of specular reflection.

### **SR-PSFs of Sample Papers**

The SR-PSFs of five kinds of sample papers and black glass are measured. Table I lists the samples used in this article. The ISO 75° specular gloss values are denoted in the table. The sample papers are ink jet paper and four coated papers (Nos. 1–4), each having different gloss. The ink jet paper is photo-grade, resin-coated paper. The gloss values of the four coated papers are changed by the number of calendaring treatments. The base is A2-grade coated paper. The basis weight is approximately 127.9 g/m<sup>2</sup>. The SR-PSF of the sample is measured along the machine direction of the paper.

Figures 9 and 10 show the measured SR-PSFs by using the apparatus. The x and y axes describe the pixel position (x', y') on the CCD camera. The intensity (z axis) describes the output value of the CCD camera. Fig. 9 shows typical measurement results. The shape of the SR-PSF of black glass is almost the same size as the original pinhole. This result shows that this system can measure the SR-PSF with high accuracy. In contrast, as shown in Fig. 10, the SR-PSF of coated paper No. 1 has a broad distribution. The measured SR-PSFs of the coated paper samples have a broad distribution corresponding to the decreasing ISO 75° specular gloss of the sample papers. That is, the peak values of the SR-PSFs decrease with the decreasing values of ISO 75° specular gloss.

The intensity was measured linearly as the value compared with the reference. Therefore the intensity of the coated paper sample used only a part of the available dynamic range. It would be able to use a wide dynamic range by using a suitable reference. The graphs were drawn linearly to indicate the differences of the coated paper samples. A graph with a logarithm scale would be able to improve evaluation visually for the different category samples.

The ISO  $75^{\circ}$  specular gloss values are almost equal in the ink jet paper and coated paper No. 1 samples. However, their SR-PSFs are considerably different in Fig. 10(b1) and (b2). In this case, the visual gloss is not in agreement with that of the ISO specular gloss. The reflected light image on the ink jet paper is much sharper than that on coated paper



Figure 11. Schematic diagram of the experiment in which the reflection image is captured.

No. 1 in the visual test of sharpness shown in Fig. 4. The ink jet paper visual gloss is also higher than that of coated paper No. 1. The subjective evaluation value of the visual gloss is dependent on the SR-PSF distribution. The same result regarding the subjective evaluation was reported by one of the authors.<sup>8,9</sup> Leloup et al. reported on a psychophysical study with real stimuli that are different regarding multiple visual gloss criteria.<sup>12</sup>

# Confirmation that SR-PSF is a Transfer Function of Specular Reflection Image

In an experiment, it was confirmed that the proposed SR-PSF works as a transfer function of the specular reflection image. All of the SR-PSF distributions of the paper samples have a characteristic shape. The SR-PSF distribution is broad with respect to the incidence plane and narrow in the cross direction. Since it is assumed that the SR-PSF is the transfer function of the reflection image system, the reflection image should be spread wider (i.e., out of focus) with respect to the incidence plane. In the following paragraphs, we confirm this by an experiment using sample images.

Figure 11 shows a schematic diagram of the experiment in which the reflection image is captured by the CCD camera with a normal lens system. Therefore, a sample surface like a mirror should be finely reflected to the CCD camera.

The first images captured by the CCD camera are the sinusoidal images, as shown in Figure 12. Figure 13 shows the results of the Chinese character image. These experimental results show that the reflection image has low resolution with respect to the incidence plane, and high resolution in the cross direction. The sharpness of the image is also in good agreement with the spread condition of the SR-PSF. These results are also consistent with the features of the conventional PSF. It was confirmed that the SR-PSF works as a transfer function for a specular reflection image. In previous work, the authors reported that the paper gloss image can be simulated by using this SR-PSF as a transfer function.<sup>13</sup>

## Relation Between the Reflection Angle and the Measurement Position

It was confirmed that the reflection angle of light can be measured by the apparatus. As mentioned above, the proposed apparatus for SR-PSF measurement consists of the collimator optical system. In the collimator optical system,





Figure 12. Captured image results of the sinusoidal images. The center graphs show the SR-PSF.



Figure 13. Captured image results of the Chinese character image. The center graphs show the SR-PSF.

the focal position is determined by the angle of light, as shown in Fig. 6. We investigated experimentally the relation between the reflection angle and the measurement position in this apparatus. The sample bed was modified to enable rotation, as shown in Figure 14. The reflection angle changes by rotating the sample bed arbitrarily. The detected position on the CCD was measured in the experiment. Black glass was used as a

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Figure 14. Sample bed with rotation system (yawing system). (a) Photograph showing the sample bed. (b) Schematic diagram of rotation.



Figure 15. Measured and calculated results of the observed position corresponding to the changing sample bed deviation angles,  $\Delta \theta_x$  and  $\Delta \theta_y$ . The observation angle,  $\theta_{view}$ , is 75°.

sample because the SR-PSF of black glass becomes a very small point.

Figure 15 shows the measured position corresponding to the rotation angle,  $\Delta \theta_x$  and  $\Delta \theta_y$ . The dots in the figure show the measured positions corresponding to the rotation angle  $(\Delta \theta_x, \Delta \theta_y)$ . The graph has 25 dots. The center dot shows the position for the rotation angle (0, 0). The rotation angles,  $\Delta \theta_x$  and  $\Delta \theta_y$ , were varied among  $-2^\circ$ ,  $-1^\circ$ ,  $0^\circ$ ,  $+1^\circ$  and  $+2^\circ$  to an accuracy of 0.1°. Here, the change of the reflection angle of light becomes twice the change of the rotation angle of the sample bed.

We used a mathematical model to verify the experimental results. Furthermore, we used the mathematical model to carry out a simulation under various conditions. Figure 16 shows the coordinate system in this article. The reflected



Figure 16. Coordinate system of measurement apparatus for the SR-PSF.

light is observed at the position (x, y, z). The direction with respect to the incidence plane is y, the cross direction is x, and the perpendicular axis of the x-y plane is z. The focal length, f, is the distance from the origin and is the radius of the focus sphere. The focal length is 50.1 mm in this experiment. The deviation position of the reflected light by changed angles  $\Delta \theta_x$  and  $\Delta \theta_y$  is calculated according to Eq. (2). Here,  $\theta_{view}$ is the observation angle. It is also the CCD camera angle. In this experiment,  $\theta_{view}$  is fixed at 75°.

The CCD camera has a two-dimensional array. The reflected light, the point image, is a pixel on the CCD camera. Therefore, the new coordinate system (x', y', z') corresponding to the CCD camera should be set. The CCD camera pixel position (x', y') is calculated by rotation of the vector in Eq. (3).

$$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = f \begin{pmatrix} \sin(2\Delta\theta_x)\cos(\theta_{view} + 2\Delta\theta_y) \\ \sin(\theta_{view} + 2\Delta\theta_y) \\ \cos(2\Delta\theta_x)\cos(\theta_{view} + 2\Delta\theta_y) \end{pmatrix}.$$
 (2)

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Figure 17. SR-PSF (image) of coated paper No. 1 sample and the calculated observed position (dots) corresponding to the changing sample bed deviation angles,  $\Delta \theta_x$  and  $\Delta \theta_y$ . The observation angle,  $\theta_{view}$ , is 75°.

$$\begin{pmatrix} x'\\ y'\\ z' \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0\\ 0 & \cos(\theta_{view}) & \sin(\theta_{view})\\ 0 & -\sin(\theta_{view}) & \cos(\theta_{view}) \end{pmatrix} \begin{pmatrix} x\\ y\\ z \end{pmatrix}.$$
 (3)

The experimental results were verified by the mathematical model in Eqs. (2) and (3). Figures 15 and 17 show the calculated positions of changing the sample bed rotation angles  $\Delta \theta_x$  and  $\Delta \theta_y$ . The measured results agree well with the calculated results in Fig. 15. It is clear that the observed position in this apparatus shows the reflection angle. From these results, we found that this apparatus functions as a goniophotometer within a narrow solid angle.

### DISCUSSION

### The Meaning of the Value Measured by the Proposed Measurement System

The apparatus was developed based on the measurement principle of the conventional PSF in order to measure the SR-PSF. The SR-PSFs of the sample papers showed a certain distribution and thus the transfer function of the specular reflection image. On the other hand, it was confirmed that the detected position corresponds to an angle of reflection. The intensity of light at each position is the quantity of light reflected at a certain angle from the surface. Therefore, the measured distribution is the angle-dependent distribution of light reflected from the surface. This is the gonio-reflectance distribution. It is concluded that the measured distribution of this apparatus is the SR-PSF, and it is a gonio-reflectance distribution.

Elton and Preston reported on polarized light reflectometry.<sup>14,15</sup> Obein et al. measured the shape of the specular peak of the sample in the restricted field of a very glossy surface.<sup>16</sup> These are similar systems which can measure the reflected angle and intensity at the same time. Since these techniques are constructed with the two-dimensional CCD imaging system, the two-dimensional gonio-reflectance distribution can be measured in one operation.

### The Reason Why SR-PSF Becomes an Oval Distribution

In this experiment, the measured SR-PSF became an oval distribution. Fig. 17 shows the SR-PSF of coated paper No. 1 and the reflection deviation angles. The angles were  $\Delta \theta_x$  and  $\Delta \theta_y$ , which were changed by  $-2^\circ$ ,  $-1^\circ$ ,  $0^\circ$ ,  $+1^\circ$  and  $+2^\circ$ . Here, the change of the reflection angle of light becomes twice the change of the surface deviation angle. The observation angle,  $\theta_{view}$ , the specular reflection angle, was fixed at 75° in Fig. 17. The results show that the detected positions become an oval distribution according to the isotropic reflection angle changes.

A simulation was carried out under the condition of a different observation angle,  $\theta_{view}$ , by using the mathematical model. The angles were 0°, 45°, 60°, 75° and 90°. These results are shown in Figure 18. The observed position distribution in relation to the reflection angle deviation is changed by the specular reflection angle. The reflection angle distribution becomes isotropic, particularly when the specular reflection angle is 0°. It is found that the aspect ratio results from the observation angle. Even if the surface normal distribution is isotropic, the SR-PSF is observed as an oval distribution in the case that the specular reflection angle is not zero. Thus, the reason why the SR-PSF distribution becomes an oval is based on the geometry of the observation angle. The surface normal distribution is considered isotropic in general.

This result can explain the phenomena of reflection. For example, in the scenery of a wood or a building reflected to the water surface, the reflected image fades much more in the lengthwise direction than in the transverse direction. This is because the SR-PSF is generally spread with respect to the incidence plane rather than in the cross direction.

### Surface Coarseness and Gonio-reflectance Distribution

If it is assumed that the reflection angle is determined by only the surface normal, the surface normal distribution can be calculated from the gonio-reflectance distribution by the mathematical model. The surface normal distribution shows the information of the geometry of the physical surface. Many reflection models in computer graphics have used this information as the BRDE.<sup>17–19</sup> The current authors studied a computer graphics model using the surface normal distribution measured by this apparatus.<sup>20</sup>

The surface normal distribution also correlates with the surface coarseness. The relation between surface coarseness and gloss has already been reported in research on the gloss of paper.<sup>21–23</sup> In the future, the relation between surface coarseness and gonio-reflectance distribution will also be clarified.

#### The Merits of this Measuring Technique

This study could benefit industry in terms of the development of a standard measurement instrument. It seems that the SR-PSF is correlated with the visual gloss. The SR-MTF can be converted from the SR-PSF by Fourier transform. The DOI value at the destination spatial frequency could be estimated easily by using the SR-MTF. The measuring



Figure 18. Calculated position (dots) corresponding to the changing sample bed deviation angles,  $\Delta \theta_x$  and  $\Delta \theta_y$ . The different observation angles,  $\theta_{\text{view}}$ , from (a) to (e) are 0°, 45°, 60°, 75° and 90°, respectively.

technique can be used to further develop a high-resolution goniophotometer. Since the technique measures two dimensions with the CCD camera, the two-dimensional gonio-reflectance distribution can be measured in one operation. The angular resolution is high because the CCD camera consists of many pixels. However, the range of the angles that can be measured at this time is not wide. This apparatus functions as a high-resolution goniophotometer within a narrow solid angle.

### CONCLUSION

We proposed the SR-PSF. We developed an apparatus to measure the SR-PSF. The experimental results showed that the measured SR-PSF works as a transfer function for the specular reflection image. We also showed that the apparatus works as a goniophotometer within a narrow solid angle. The measured data are the SR-PSF with a gonio-reflectance distribution. The reflection angle is caused by the surface normal. The surface normal distribution can be calculated from the gonio-reflectance distribution by the mathematical model. The SR-PSF distribution is oval because it is based on the geometry of the observation angle. We are going to develop an evaluation technique for the visual gloss and enhance the high-resolution goniophotometer by this measuring technique.

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