# Fast Estimation Algorithm for Calculation of Reflectance Map based on Wiener Estimation Technique

Koichi Takase, Norimichi Tsumura, Toshiya Nakaguchi and Yoichi Miyake

Department of Information and Image Sciences, Chiba University, 1-33, Yayoi-cho, Inage-ku, Chiba 263-8522, Japan

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We propose a fast estimation algorithm for calculation of a reflectance map from measured radiances based on Wiener estimation technique. This technique uses many sample data of reflectance maps to decide transformation for accurate estimation. Reflectance maps of objects are estimated rapidly by linear operation between a vector of measured radiances and an estimation matrix previously calculated based on the sample data.

**Key words**: reflectance map, Wiener estimation, BRDF, light field, linear operation

E-mail address: takase@graduate.chiba-u.jp

### 1. Introduction

With the development of the digital imaging system, a digital archiving system has been used in museums to preserve the collections and exhibit them by digital display technology. For this archiving, reflectance maps of objects must be recorded to accurately preserve the reflectance property of the objects. <sup>1)</sup> The reflectance map of an object is a function of angle for incident light and it denotes a ratio between radiance of reflected lights from objects and radiance of incident lights to objects. The radiance to outgoing angle is measured against incident lights from many directions as shown in Fig. 1. The reflectance maps of objects can be calculated by fitting the bidirectional reflectance distribution function (BRDF) models such as Phong model<sup>2)</sup> and Ward model<sup>3)</sup> to the measured radiances. However, it takes much time to fit the model to the measured radiances at each point of the object surface, since this fitting operation needs nonlinear iterations. Practically, a light source cannot be assumed as a distant point light source, because the distance between objects and light source is not usually long enough and size of light source cannot be ignored. Therefore, radiances of incident lights should be described by the distribution of radiance rays (light field<sup>4, 5)</sup> from the light source for accurate estimation of a reflectance map. If the light field of the light source is considered, much more time is needed for calculation by the model fitting.

In this paper, we propose a fast estimation algorithm for calculation of a reflectance map from the measured radiances based on the Wiener estimation technique. The estimation technique determines an estimation matrix based on the minimum mean square error criterion using sample data of reflectance maps. The proposed method can estimate a reflectance map of an object rapidly by linear operation between the vector of measured radiances and the estimation matrix. In this method, the size of the light source and distance between the light source and an object does not influence duration time for calculation in estimation of the reflectance map.

In the next section, details of the proposed method are described. In § 3, accuracy and duration time for estimation by the proposed method are evaluated by computer simulation. The experiments that were performed to evaluate the method are presented in § 4. In that section, accuracy of the method is evaluated by changing a number of measured data for estimation and sample data for estimation matrix.

# 2. Proposed Method

Figure 2 shows the flow of the process in the proposed method. We define  $f_1$  as incident radiance vector for light source at position #1. The vector  $f_1$  is a  $l \times 1$  column vector, where each element of the vector indicates the incident radiance at each angle of incident light. Figure 3 shows an example of incident radiance vector (a broken line). The horizontal axis indicates angle of incident light to the object and the vertical axis indicates radiance of incident ray. The radiance is normalized by the maximum value of incident radiances in this figure.

Let us define  $F = [f_1, f_2, ..., f_m]^t$  as an incident radiance matrix for m positions of light source, **n** as additive noise of a CCD camera which is signal independent, and  $\rho_s$  as sample data of reflectance maps which is calculated from a BRDF model with valid model parameters. Figure 3 also shows an example data of a reflectance map  $\rho_s$  (a solid line). The reflectance is normalized by the maximum value of reflectance at each angle of incident light for illustration in this figure. Let us suppose that the matrix F has already been measured by preprocessing. Measured radiance of reflected lights  $v_s$  is written by the following equation:

$$\boldsymbol{v}_s = F\boldsymbol{\rho}_s + \boldsymbol{n} \,. \tag{1}$$

We will consider an estimation matrix *G* which estimates reflectance map  $\hat{\rho}$  from measured radiances of reflected lights *v* as

$$\hat{\boldsymbol{\rho}} = \boldsymbol{G}\boldsymbol{v} \,. \tag{2}$$

The matrix G in the Wiener estimation is determined to minimize an ensemble average of square error between  $\rho_s$  and the estimated reflectance map for all sample data. The matrix G is written as follows.

$$G = R_{\rho\rho} F^{t} \left( F R_{\rho\rho} F^{t} + R_{nn} \right), \tag{3}$$

where,

$$R_{\rho\rho} = \langle \boldsymbol{\rho}_{s} \boldsymbol{\rho}_{s}^{t} \rangle, R_{nn} = \langle \boldsymbol{n} \boldsymbol{n}^{t} \rangle.$$
(4)

The operation  $\langle \rangle$  denotes the ensemble average for sample data.

In the proposed method, the reflectance map of objects  $\hat{\rho}$  is estimated rapidly by a linear operation between the measured radiances of reflected lights and the estimation matrix.

# 3. Evaluation of the Proposed Method by Computer Simulation

#### 3.1 Process of computer simulation

Accuracy and duration time required for the estimation are evaluated by computer simulation. The proposed and conventional methods are used to estimate 120 test patterns of reflectance maps. The test patterns are calculated with Ward model<sup>3)</sup> and model parameters of several reflectance properties. Ward model is described as follows.

$$\rho(\theta_i, \phi_i, \theta_o, \phi_o) = \frac{\rho_d}{\pi} + \rho_s \frac{\exp\left[-\tan^2 \delta \left(\cos^2 \phi_i / \alpha_x^2 + \cos^2 \phi_i / \alpha_y^2\right)\right]}{4\pi \alpha_x \alpha_y \sqrt{\cos \theta_i \cos \theta_o}}, \quad (5)$$

where,  $(\theta_i, \phi_i)$  and  $(\theta_o, \phi_o)$  denote the direction of incident light and the direction of the viewer,  $\rho_d$  and  $\rho_s$  represent the diffuse and specular coefficients, respectively, and  $\alpha$  is surface roughness,  $\delta$  denotes angle between surface normal and half vector which is between directions of the incident and reflected radiances. The model parameters are selected randomly in a valid range of Ward model parameters for this computer simulation. Test patterns of reflectance map are shown in Fig. 4. The computer simulation was performed under the following three conditions. In conditions 1 and 2, the reflectance map is estimated by the conventional method where the model of reflectance maps is fitted into the data. In condition 3, the reflectance map can be estimated by the proposed method. The light source is assumed to be a distant point light source under condition 1, or a broad light source under conditions 2 and 3. In the proposed method, sixteen sample data of reflectance maps which are calculated by Ward model are used to calculate the estimation matrix G.

#### 3.2 Results of computer simulation

Table I shows the duration time for estimation and error of the estimation. The estimation error for each reflectance map was evaluated by the root-mean-square error (RMSE) defined by

$$RMSE = \sqrt{\frac{1}{n} \sum_{i=1}^{n} (\rho_i - \hat{\rho}_i)^2} .$$
 (6)

where *n* is the number of sampling angle of incident light which is 90 in this study,  $\rho_i$  and  $\hat{\rho}_i$  represent the test pattern of reflectance map and the estimated reflectance map at the *i*th angle of incident light, respectively. A MATLAB optimization toolbox is used for the conventional methods to fit the model into the test pattern.

It took longer under condition 1 to estimate the reflectance map than under condition 2, because the broad light source increased the measured data which was used for the fitting operation. The reflectance map under condition 3 was estimated to be about 10<sup>6</sup> times as fast as that under condition 2, since reflectance map can be estimated by linear operation under condition 3. The RMSE for condition 2 was smaller than that for condition 1, because light field is considered under the former. The RMSE for condition 3 was also smaller than that for condition 2. To see the difference in detail, we compared the reflectance map of each condition. Figure 5 shows magnified parts of the map, where estimation error is large in the case of the largest RMSE of the proposed method. This figure shows that the reflectance map can be estimated by the proposed method almost as accurately as by the conventional method.

#### 4. Experiments

#### 4.1 Experimental environment

Reflectance maps of four real objects were measured to evaluate the accuracy of the proposed method. Figure 6 shows the system for measurement of a reflectance map. The objects are illuminated by LED from an arbitrary direction and reflected radiance is measured by a spectro-radiometer (CS1000 KONICA MINOLTA) in the system for measurement. We can assume that the LED is a point light source in this experiment, since size of the LED is small enough and we need to evaluate only the estimation performance of the proposed method.

In the measurement, the real objects were illuminated from an angle of 10 degrees to 80 degrees at 1 degree intervals where surface normal was defined as an angle of 0 degrees. Reflected radiance to angle of -45 degrees was measured by the spectro-radiometer. Using Blinn-Phong model, <sup>6)</sup> reflectance maps were estimated by changing the number of angles for incident lights in the estimation. Blinn-Phong model is described using the following vector notation.

$$\rho = k_d \left( \boldsymbol{N} \cdot \boldsymbol{L} \right) + k_s \left( \boldsymbol{N} \cdot \boldsymbol{H} \right)^n, \tag{7}$$

where N and L denote surface normal and direction of incident light, respectively,  $k_d$  and  $k_s$  represent the diffuse and specular coefficient, respectively, n is surface roughness. H is described as H = (V + L)/|V + L|, where V denotes the direction of the viewer. Table II shows the number of the measured data for estimation and angles of incident lights for the measured data. Table III shows the range of the model parameters for sample data which are used to calculate the estimation matrix in the proposed method.

#### 4.2 Experimental results

Accuracy of the proposed method is evaluated based on the experimental data in this section. As the first step of the evaluation, the number of measured data for estimation is changed in the experiments. The RMSEs of the conventional and the proposed methods at each number of measured data are shown in Table IV. The estimation accuracies of the reflectance maps for acrylic resin, plastic and tile were the same by the two methods. However, the estimation accuracy of the reflectance map for styrene plastic by the proposed method was low compared with the conventional method, because the reflectance property of styrene plastic is different from that of the other objects measured, as shown in Fig. 7. When a few measured data were used for estimation, the accuracy of the estimation was low by the conventional method. In contrast, we can see that the accuracy of the estimation was better in the proposed method, because the reflectance properties of the sample data were available. However, when the reflectance properties of the measured objects and the sample data do not match each other, the accuracy of the estimation will be lower as we see in the estimation of the reflectance map for styrene plastic. To evaluate the influence of the reflectance properties of the sample data for estimation, we changed the

sample data used in the preprocessing to reduce the estimation error in the case of styrene plastic. The new sample data are calculated in the range of model parameters as shown in Table III. The RMSEs of the estimation are shown in Table V. We can see that the estimation was highly accurate at each number of measured data. From this evaluation, we can see that sample data which have similar reflectance properties to the measured objects must be selected for use in the proposed method.

As the second step, accuracy of the proposed method is evaluated by changing the number of sample data to calculate the estimation matrix. We changed the number of sample data whose parameters are in the range shown in Table III. Table VI shows five conditions of sample data for evaluating the accuracies by changing the variation of model parameters in sample data. Table VII shows the RMSEs of the estimation for reflectance maps of objects for each piece of sample data in Table VI. The accuracy of the estimation is reduced as the number of sample data is reduced. This tendency is the same in the case of acrylic resin, plastic and tile. The accuracy of the proposed method depends on the number of data samples. From Tables VI and VII, we can see that the parameter of surface roughness greatly influenced the accuracy of the estimation compared with other parameters. It is difficult to estimate a reflectance map from sample data calculated from a few surface roughnesses, because the model parameter influences reflectance maps with highly non-linear property. However, the accuracy of the estimation does not depend on the number of sample data in the case of styrene plastic. The sample data for the estimation matrix have

reflectance properties similar to styrene plastic, because the range of the parameters is small as shown in Table III. Therefore the number of sample data did not influence the accuracy of the estimation.

Finally, we evaluated duration time for the estimation of the conventional and the proposed methods, and found it to be 160 ms and 0.02 ms, respectively. The reflectance was estimated by the proposed method eight thousand times faster than by the conventional method.

# 5. Conclusion and Discussion

We estimated the reflectance maps of objects rapidly using the Wiener estimation technique. The accuracy and duration time for estimation were evaluated by computer simulation. Influence of the number of sample data and measured data was also evaluated by the experiment of the estimation for the reflectance map. The proposed method depends on sample data of the reflectance map to calculate the estimation matrix. Sample data which have reflectance properties similar to the objects to be measured must be selected in the proposed method.

In the case of reflectance map estimation of three dimensional objects, surface normals of the objects face in several directions, so that we need sample data of the reflectance map which face each direction. One solution to this problem is that sample data facing any direction are stored in the sample database. Since the volume of computer memory is increasing, all sample data can be stored in the computer. However, duration time required to search for sample data should be considered in a practical experiment.

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Another solution is to rotate specular directions to 0 degrees by rotating the reflectance maps as done by Wood et al.<sup>7)</sup> In this case, we need only sample data whose direction of specular reflection is at an angle of 0 degrees. However, estimation error may increase at an angle of incident lights which is far from the specular angle. In our future work, we will reproduce three dimensional objects by the proposed method in the above manner.

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# Captions of figures

Fig.1. Measurement for reflectance property of an object.

Fig.2. Flow diagram of the process of proposed method.

Fig.3. Incident light vector and reflectance map of an object.

Fig.4. Test patterns of reflectance map.

Fig.5. Magnified parts of reflectance maps, where estimation error is large in the case of the largest RMSE of proposed method.

Fig.6. Measurement system of reflectance map.

Fig.7. Reflectance maps of measured objects.

Condition	Method	Light source	Duration time (ms)	Estimation error (RMSE)
1	Conventional	Distant	2070	0. 633
2	Conventional	Broad	13800	0. 073
3	Proposed	Broad	0.015	0. 179

Table I. Duration time for estimation and error of estimation under three conditions.

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Number of measured data	Angles of incident lights (degree)
2	44, 45
5	43, 44, 45, 46, 47
7	42, 43, 44, 45, 46, 47, 48
9	41, 42, 43, 44, 45, 46, 47, 48, 49

Table II. Number of measured data for estimation and angles of incident lights for measured data.

Table III. Range of model parameters for sample data used to calculate estimation matrix in the proposed method.

	Coefficient of	Coefficient of	Coefficient of			
	diffuse reflection	specular reflection	surface roughness			
Range for	0 150~0 318	0.5~100	10000~50000			
all objects	0.139 0.318	0.5 100	10000 - 30000			
Range for	0 150~0 318	0.5~1	$100 \sim 1000$			
styrene plastic	0.137 0.318	0.5 °1	100 91000			

Object	Mathad	The number of measured data				
Object	Method	2	3	5	7	9
A organia regin	Conventional	15.8	15.7	2.23	2.23	2.22
Actylic resili	Proposed	3.22	2.24	2.22	2.22	2.22
	Conventional	20.2	16.1	0.652	0.644	0.639
Flastic	Proposed	1.20	0.798	0.646	0.631	0.636
Tila	Conventional	12.0	10.1	6.85	0.53	0.500
The	Proposed	0.708	0.470	0.464	0.463	0.464
Styrene plastic	Conventional	0.130	0.332	0.0670	0.124	0.123
	Proposed	0.283	0.284	0.271	0.0975	0.221

Table IV. RMSEs of the conventional and the proposed methods at each number of measured data.

Object	Mathad	The number of measured data					
Object	Method	2	3	5	7	9	
Styrono plastia	Conventional	0.130	0.332	0.0670	0.124	0.123	
Styrene plastic	Proposed	0.0940	0.114	0.142	0.0985	0.0564	

Table V. RMSEs of the estimation for reflectance map of styrene plastic with new estimation matrix.

Condition	А	В	С	D	E	
Number of sample data	125	50	50	50	8	
Number of parameters for diffuse reflection	5	2	5	5	2	
Number of parameters for specular reflection	5	5	2	5	2	
Number of parameters for surface roughness	5	5	5	2	2	

Table VI. Number of model parameters for evaluation of estimation accuracies.

Condition	А	В	С	D	Е		
Acrylic resin	2.22	2.23	2.22	2.45	2.67		
Plastic	0.636	0.640	0.637	0.755	0.795		
Tile	0.464	0.467	0.464	0.546	0.522		
Styrene plastic	0.0564	0.0559	0.0556	0.0585	0.0574		

Table VII. RMSEs of the estimation for reflectance map of objects for each sample in Table VI.